PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
Volume 20 August 15, 1934 Number 8

PROPOSAL OF A METHOD OF OBSERVING THE SOLAR CORONA WITHOUT AN ECLIPSE

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Read before the Academy, Monday, April 23, 1934

A number of times in the past, progress in some science or art has advanced to the state where it becomes useful in some other unrelated field for a purpose different from that for which it was primarily developed. It is the purpose of this paper to suggest that the technique of television has been developed to the point where it apparently can be employed to advantage for a study of the solar corona at times other than those of eclipses. Such a use would take advantage of the means of image discrimination which this technique offers but not the ability to transmit images over appreciable distances. The use of this newly developed technique for image discrimination, particularly to increase contrast, is not, of course, limited to the specific application mentioned here. It might be used advantageously in other fields as well.

The problem of observing or photographing the details of the corona without an eclipse has challenged the efforts of able investigators for the past fifty years, among whom have been such men as Huggins, Hale, Ricco, Deslandres, Wood, Hansley and, lately, Lyot. The last named was the only one to achieve any degree of success. On a day of remarkable atmospheric transparency with his apparatus at the top of Mt. Pic du Midi, Lyot[1] was able to obtain images which showed several faint irregularities of the light around the sun's disc. These apparently were coronal features. The excellence of his work and apparatus leaves little to be hoped for in the way of improvement. Furthermore, his method gives no promise for any but the innermost part of the corona. For adequate solution the problem demands a method which will separate adequately the feeble light of the corona not only from the diffuse light in the telescope but also from the brilliant glare of the earth's atmosphere around the solar disc. We therefore need to attack the problem along entirely new lines.

Enough has been learned by previous investigators so that the factors of the problem seem to be well in hand. Lyot[2] has found that the brightness

of the atmospheric glare 20 minutes from the limb of the sun varies from 5 to 50 millionths of the surface brightness of the sun's disc, depending on conditions of the sky and altitude of the observer. The brightness of the corona as measured by Abbot at the eclipse of 1908 was 1.3 millionths at 1.5 minutes from the solar limb and 0.4 millionths at 4 minutes. Taking Bergstrand's law that the intensity varies inversely as the square of the distance from the limb, the intensity at a distance of 20 minutes would be 3.2×10^{-3} millionths. We need, therefore, a method which will discriminate sufficiently in favor of the feeble light of the corona to make it visible against the background of atmospheric glare which at this distance (about two-thirds of the solar diameter) is a thousand times as bright. No ordinary photographic or optical method gives any promise of this. Of course for the innermost corona the above ratio is not nearly so great and much less discrimination is needed.

In the mechanism of television an image is converted by rapid scanning, into an electric current which varies in accordance with the various features of the image. Let us suppose that the image of the sky around the sun's disc is scanned by a small opening behind which a photoelectric cell is placed and that the scanning is done in a spiral path around the sun starting from the limb. As the spot passes over the hoods, arches and streamers of the coronal image the photoelectric current will consist of a number of components of different frequencies. Now if we pass this current through appropriate electrical filters we may eliminate the direct current component due to skylight and telescopic glare as well as the low frequency components which arise as a result of the radial progression of the scanning hole, leaving, presumably, only the high frequency components due to the corona and inhomogeneities caused by poor seeing. (Such inhomogeneities would, of course, be continually varying and would thus be no more detrimental to observation in this case than in any other sort of astronomical observation.) This would appear to present no great difficulty even though the ratio of intensities is of the order of hundreds of times, since the frequencies are widely separated. Then in the usual way the image would be reconstructed from the current. This new image would now contain the details of the corona without any skylight glare. If bright stars are in the field they would shine out as dots of light the shape of the scanning hole. We should thus be able to reproduce the major features of a solar eclipse at any time when the sky is clear.

The faintness of the outer corona will set a limit on the size of the scanning hole which may be used with a telescope of given aperture and focal length. Consider a telescope of 10 cm. aperture which gives a solar image 10 cm. in diameter. The amount of light received from the solar image by a scanning hole 2 mm. square will be 0.54 lumen. The amount of coronal light at a distance of 20 minutes will be

$$0.54 \times 3.2 \times 10^{-9} = 1.7 \times 10^{-9} \text{ lumens.}$$

At a distance of 1.5 minutes it will be equal to 0.7×10^{-6} lumens. A photoelectric cell has a sensitivity of the order of 100 microamperes per lumen. Thus the coronal currents would be 1.7×10^{-13} amperes at 20 minutes and 0.7×10^{-10} amperes at 1.5 minutes.

The photoelectric currents used in the Bell Telephone Laboratories' demonstration[3] of television in 1930 were of the order of 10^{-10} amperes, so that with the above set-up, if this is taken as the lower limit of usable photoelectric currents, only the inner corona would be visible.

This size of the scanning hole corresponds to about 36 seconds square. For the outer corona a larger hole could be advantageously used to show up the great streamers. For a telescope of larger size with, let us say, a 100 cm. objective, the light gathering power would be increased 100-fold and coronal details out to about 10 minutes might then be made visible. For a 100-inch telescope this theoretical limit would be moved out to 20 minutes and by increasing the size of the scanning hole the general form of the long streamers could probably be observed.

As Lyot[4] has shown, the diffused light from a telescope may be as intense as that of the atmosphere and he has demonstrated the means by which it may be reduced if necessary. It should, therefore, cause no real trouble.

An advantage of this method over photography as used at eclipses would be that the average intensities of the inner and outer corona could be made equal so that the details of both might be observed at once. The device by which we might seek to accomplish this is the familiar automatic volume control used on radio receiving sets.

Two general types of apparatus are possible, a set-up similar to the usual television system allowing a continuous watch of the coronal features and one similar to a facsimile or telephoto arrangement giving a reproduction of the coronal image on photographic paper. These might be called an electrocoronascope and an electrocoronagraph, respectively. The latter would be somewhat easier of realization since the scanning speed is much slower. It would probably be well to allow the photoelectric cell to rotate with the scanning opening in order to avoid spurious images due to irregularities on various parts of the photo-cell. Problems such as this are a matter of technique and do not seem to present insuperable difficulties.

The lower limits of the intensities of light used in the above calculations are probably higher than necessary, in view of the recent advances in photo-cell technique. Especially is this true for the electrocoronagraph set-up in which smaller intensities would be practical not only for this reason but also because of the slower scanning speed. This would mean, of course, that smaller telescopes could be used.

At the suggestion of Dr. Herbert E. Ives of these Laboratories the method was given a test in his laboratory, simulating the assumed conditions as nearly as is possible with available designs of television apparatus. A lantern slide of a coronal photograph taken at the eclipse of 1908 was imaged upon a 72-hole television scanning disc, using an illumination sufficient to give an image brightness of the order of magnitude considered in the preceding discussion. The size of the scanning holes corresponded to about 1 minute square, the whole field was a rectangle of approximately $1^1/_2° \times 1^1/_2°$ under which conditions an excellent image of the moon's disc and the corona was rendered at the receiving apparatus.

In order to test the effect of the superposition of a general glare of light, the expedient was adopted of flooding the photoelectric cell behind the scanning disc, with light directly, whereby a very great ratio of steady to intermittent photoelectric signal was obtained. This flooding light was increased until the occurrence of electrical "noise," which causes a speckled or scintillating appearance, became great enough to spoil the discrimination of the coronal features. In the experiment as performed, the flooding light could be increased to about 10,000 times the normal television level before discrimination was seriously impaired. Referring back to the figures given earlier, where the atmospheric glare was set at 1000 times the coronal brightness, it is seen that this experiment indicates the method to be adequate for producing the required discrimination.

It must be granted, however, that the conditions of this experiment were somewhat more favorable than could be expected in an actual test, because of the method used in introducing the glare. Under actual conditions the background glare would also be scanned by the disc or equivalent device. Consequently any irregularities of field intensity or structure introduced by the image forming and scanning apparatus, even though quite minute, would generate signals whose intensity would be conditioned by the level of the general glare, and so could easily swamp the coronal image. This emphasizes the necessity for using image forming and scanning arrangements substantially free from irregularities and discontinuities. Scanning arrangements meeting the requirements appear, however, to be entirely possible of construction.

The author is aware that an actual trial in the observatory may uncover unforeseen difficulties. It is felt, however, that the proposal has sufficient promise to justify a trial.

[1] *Zeit. für Astrophys.*, **5**, 73 (1932).
[2] *Compt. rend.*, **195**, 21 (1932).
[3] Ives, *Bell Laboratories Record*, **8**, 9, 401, May (1930).
[4] Loc. cit.

THE INFLUENCE OF PREGNANCY UPON THE TITRE OF IMMUNE (BLOOD-GROUP) ANTIBODIES IN THE RABBIT

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Communicated July 12, 1934

We have previously described two agglutinogens, H_1 and H_2, which may be found in normal rabbit red-blood cells. These agglutinogens are determined genetically by allelomorphs or alternative forms of the same gene, and each, when acting alone, behaves in heredity as a simple dominant unit-character. Each of the corresponding agglutinins (h_1 and h_2) is a specific antibody produced as an immunity response when red-blood cells carrying the agglutinogen are repeatedly injected into a rabbit lacking that agglutinogen in its erythrocytes.

We have also investigated the fate of embryos which had inherited from their fathers red-cell agglutinogens, H_1 or H_2, antagonistic to the immune agglutinins, h_1 or h_2, developed in the sera of their mothers before and during pregnancy. During the course of these investigations we discovered in the agglutinin titre of pregnant rabbits certain fluctuations of greater magnitude than those which we had observed in non-pregnant rabbits. For example, rabbit No. 202, which lacked agglutinogens, was mated to an H_1 buck and injected bi-weekly with H_1H_2 blood. Six days later she possessed both h_1 and h_2 agglutinins which gave a single + reaction in a 1/1 dilution. In spite of continued injections of H_1H_2 blood, neither agglutinin could be detected upon the 20th and 29th days. Injections had been discontinued on the 25th day. Parturition took place upon the 30th day. Upon the 35th day both agglutinins h_1 and h_2 were again present and again gave a single + reaction in a 1/1 dilution. This doe was then mated to an H_1H_2 buck and bi-weekly injections were resumed. Twenty-seven days later she was found to possess neither agglutinin, as was still the case just after parturition upon the 31st day and also 6 days subsequent to parturition.

These results suggested that some factor concerned with pregnancy might be responsible for the loss of titre observed, since agglutinins produced in males were normally retained during long periods of time with little change in titre.

To determine whether the differential factor might be genetic constitution, five test matings were followed, employing three does lacking agglutinogens, mated with bucks possessing a single agglutinogen. The females were injected bi-weekly with the red cells from about 5 cc. of H_1H_2 blood, and later their sera were tested for strength of agglutinins, employing a 1/1 dilution. Injection was suspended 5 days prior to parturition. The

results of these experiments are shown in figure 1, in which strength of titre is plotted against duration of pregnancy, or the period of potential pregnancy, 31 days in duration.

In the first experiment (top of Fig. 1), it will be seen that doe A196 developed a 5+ titre of h_2 agglutinin and a 3+ titre of h_1 agglutinin, without loss of titre until injections were suspended near the close of the pseudo-pregnant period, the beginning and ending of which are indicated in the figure, each by a vertical line, for it turned out that she was not pregnant. The time at which injections were suspended is indicated by a dot just above the base line in each experiment diagrammed in figure 1.

In the second experiment (Fig. 1, next to top) the doe A5 showed during pregnancy a decrease in h_2 titre but with partial recovery of titre without

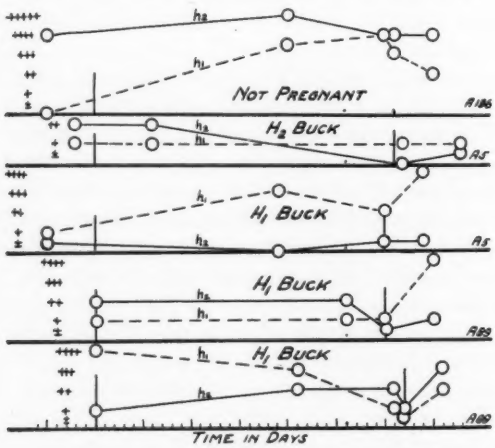

FIGURE 1

further injection following parturition. In a second mating (diagrammed immediately below the foregoing) she lost h_2 agglutinin completely but recovered it to some extent following parturition. Her h_1 titre rose during the early part of pregnancy but later dropped, only to rise again following parturition. In the fourth experiment, the doe A89 maintained titre of the two agglutinins rather constantly, with h_2 dropping at parturition, and both agglutinins showing an increase following parturition. In a second pregnancy (lowest diagram in Fig. 1) A89 lost much of her h_1 titre during pregnancy, but both agglutinins increased following parturition.

If we consider the genetic constitution of the male used in each of these matings, we find that in the first mating of A5, an H_2 buck was employed, and the titre of h_2 agglutinin was diminished. In her second mating an H_1 buck was used, and her high titre of h_1 agglutinin was reduced only to in-

crease greatly after parturition. In the first mating of A89 an H_1 buck was used, and her h_1 agglutinin showed the greatest post-parturitional increase. In her second mating also an H_1 buck was employed, and subsequently her h_1 agglutinin showed a considerable decrease in titre but recovered after parturition.

These data suggest that possibly the genetic constitution of the male, and hence that of the embryos, may have something to do with the decrease in titre of agglutinins in pregnant rabbits. But how could this be possible? We have shown elsewhere that H_1 and H_2 bucks such as we have employed in these matings will transmit to their offspring as dominant unit-characters the agglutinogens which they possess, and we have further shown that rabbit embryos develop these agglutinogens very early

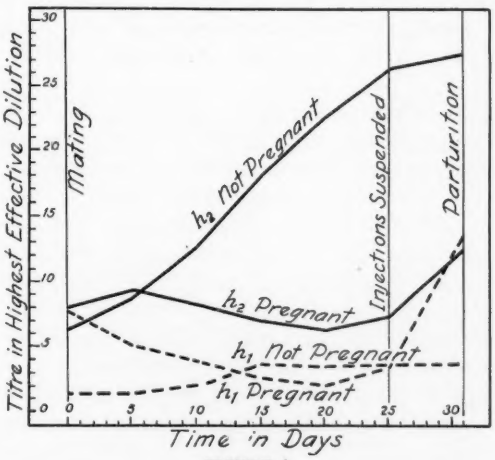

FIGURE 2

within their erythroblasts. We have also shown that these maternal agglutinins traverse the placenta and may be found in the circulation of newborn young of the appropriate genetic constitution. If the agglutinin of the mother is neutralized by the antagonistic red cells of the growing embryos in sufficient quantity, we may expect a drop in the mother's titre of *that agglutinin only* for which her embryos have inherited an antagonistic agglutinogen from their father.

Because the method which we employed of measuring increase in titre will suffice merely for weak titres where no prezone is encountered, and because only one or two points had been determined for each pregnancy, it was decided to use the same three does in further tests of the same nature but employing several refinements of technique. Titres were determined more often. Tests were made in dilution, and the highest dilution at which

a good reaction was observed was taken as a point on the curve of titre strength. Each point so determined was labeled with the strength of reaction at that dilution. This strength was usually single + but might vary up to 3+.

Eleven mating tests were made, employing the refined methods detailed above. Five of these matings proved sterile. The weaker (h_1) agglutinin was present at the time of mating in two cases only. But in all cases the h_1 titre rose during the period of potential pregnancy. The stronger (h_2) serum in these sterile matings rose greatly during the pseudo-

TABLE 1

Dilutions								
1/32	—	—	—	—	—	—	—	—
1/16	—	—	—	—	—	—	1	1
1/8	—	—	—	±	—	1	2	3
1/4	—	—	1	2	1	2	4	7
1/2	2	1 1/2	3	3	2	4	8	4
1/1	2	3	5	3	2	4	5	4
	12	15	20	2	9	14	17	23
	March			April				

Strength of reaction for h_2 in doe A5 when not pregnant.

TABLE 2

Dilutions								
1/64	—	—	—	—	—	—	—	—
1/32	—	1	—	—	—	—	—	—
1/16	1	2	1	—	—	—	—	—
1/8	3	2 1/2	3	1	±	—	—	—
1/4	3	3	3	1 1/2	2	±	—	1
1/2	4	5	5	2	3	1 1/2	1 1/2	1
1/1	6	6	6	4	3	2	3	3
	5	12	19	26	5	12	15	20
	February				March			

Strength of reaction for h_1 in doe A5 when pregnant.

pregnancies. An example of this rise is shown in table 1, in which the strength of reaction of the h_2 agglutinin at different dilutions is given for certain days before, during and after pseudo-pregnancy in doe A5.

For comparison we have shown in table 2 the strength of reaction of the same doe for h_1 agglutinin before, during and after a real pregnancy. Here it will be seen that the titre steadily decreases during pregnancy, but begins to recover after parturition. In the five sterile matings was found the same tendency to increase in titre to a maximum so long as injections continued, which tendency we had observed previously in the sterile mating of A196, recorded by earlier and less refined methods, and which we have seen also in the production of agglutinins where males and unmated females have been employed as recipients.

During the six fertile pregnancies there was observed a tendency of the titres of both agglutinins, h_1 and h_2, to decrease, especially during the second and third weeks. This lowering of titre is followed by a rise which may be evident during the last week of pregnancy (injection having been suspended) but is quite marked several days subsequent to parturition. An example of this lowering of titre during pregnancy (already referred to) is given in table 2, in which the strength of reaction of h_1 agglutinin at different dilutions is given for certain days before, during and after a true pregnancy in doe A5. This doe was pregnant by an H_1 buck, whereas A89 and A196 had been pregnant by an H_2 buck. The second pregnancy of each of these three does was by a 0 buck.

The fact that pregnancies produced by matings to 0 bucks show the same lowering of titre as matings to H_1 and H_2 bucks, proves conclusively that the lowering of titre is a phenomenon of pregnancy *per se* and is independent of the genetic constitution of the embryos being carried by the mother.

Graphs were prepared for titre strength of agglutinins, h_1 and h_2, for each of the six pregnancies and also for the five pseudo-pregnancies, plotting titre in highest effective dilution against duration of pregnancy or pseudo-pregnancy in days. From these graphs the average titre was computed for the day of mating, and subsequently for the 5th, 10th, 15th, 20th, 25th and 30th days. This procedure gave us the averages used in figure 2, in which strength of agglutinin is plotted against duration of the pregnant period. The average h_1 titre for *non-pregnant* periods *rises* from 1.3 to 3.6, and the average h_2 titre for *non-pregnant* periods *rises* from 6.5 to 27.5. The average h_1 titre for *pregnant* periods *drops* from 7.7 to 2.0 and then rises, after the suspension of injections, to 13.3. The average h_2 titre for *pregnant* periods *decreases* from 8.0 to 6.2 and then rises, after the suspension of injections, to 12.4.

Several explanations are possible. One is that the young during their development produce some substance which passes via placenta into the maternal blood stream, capable of preventing the elaboration of agglutinins. Another is that the embryo uses up some substance of the mother's blood serum which traverses the placenta and is necessary for the production of antibodies. This latter explanation seems more plausible in view of our previous findings that newborn rabbits of the appropriate genetic constitution for producing agglutinins h_1 and h_2 lack some substance necessary for their production.

In connection with our results, a note of Puccioni is of interest. This author reported a post-parturitional rise in titre of α and β normal agglutinins in women regardless of the blood group of the child.

Summary.—Female rabbits decrease markedly in their capacity to produce the immune (blood-group) antibodies, h_1 and h_2, during the second

and third weeks of pregnancy, but there follows a sharp increase in titre of the antibodies, which is especially evident during the first few days after parturition. That this increase in antibody titre is not directly the result of neutralization by incompatible embryos is shown by the fact that pregnancies in which compatible embryos only are involved, exhibit the same titre decrease.

[1] Castle and Keeler, *Proc. Nat. Acad. Sci.*, **19**, 92–100 (1933).
[2] Keeler and Castle, *Ibid.*, **19**, 403–411 (1933).
[3] *Ibid.*, **20**, 273–276 (1934).
[4] Puccioni, *Bol. Soc. Int. Microbiol.*, **3**, 686–690 (1931).

RADIOACTIVITY ARTIFICIALLY INDUCED BY NEUTRON BOMBARDMENT

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Communicated June 14, 1934

When certain substances are bombarded with deutons, many and varied nuclear reactions take place. From a target of calcium fluoride, for example,[1,2,3,4] one observes the simultaneous emission of protons, scattered deutons, alpha-particles, neutrons, particles of electronic mass and gamma rays. In addition it is found that nuclear reactions occur in which radioactive substances are produced.

The very richness of the phenomena renders interpretation difficult and it is therefore desirable to select for separate investigation those nuclear processes that can be made to evidence themselves in most direct fashion. In particular, the recent important discovery of Fermi,[5] that in several substances radioactivity is induced by neutron bombardment, raises an uncertainty in some cases as to the nuclear reactions responsible for the radioactivity induced by deutons. The question is whether the deuton activity is a primary effect of the type discovered by Curie and Joliot[6] or the result of second order processes wherein neutrons, first liberated by the deutons, subsequently react with other nuclei.

It is clear that answers to questions of this kind can be obtained from quantitative studies of both the production of neutrons by deuton bombardment and the radioactive effects induced by neutrons. As we[3] had already some roughly quantitative knowledge of the yield of neutrons in several substances, immediately upon receiving Professor Fermi's first announcement we undertook an investigation of the neutron-induced radioactivity, the first results of which are here reported.

First of all we made a cursory examination of substances readily available, to find which ones exhibit the most intense effects. Surprisingly great radioactivity was observed to be produced in calcium fluoride, aluminum and silver, in confirmation of Professor Fermi's original observations. Smaller effects were observed in silicon, copper and zinc, while no appreciable activity was detected in lithium, beryllium, boron, carbon, magnesium, sodium, chlorine, calcium, zirconium, chromium, tungsten, platinum, bismuth and lead. In these observations the substances were exposed for about five minutes to a beam of neutrons from a beryllium target bombarded by one microampere of 2.6 MV deutons. Though we do not know accurately the number of neutrons produced by the deutons, it is roughly estimated that about 5×10^8 neutrons per second emanated from the beryllium, and that a thirtieth of these (estimated from the solid angle) passed through the substances investigated. The targets were exposed to the neutrons by being placed outside the vacuum chamber near the mica window, through which in other experiments, alpha-particles and protons have been observed.[7] After exposure to the neutrons, the substances were immediately placed in front of a Geiger counter. The induced radioactivity in silver and calcium fluoride produced more than a thousand counts per minute, and in aluminum about half that many. In silicon, copper and zinc from 30 to 100 counts per minute were observed.

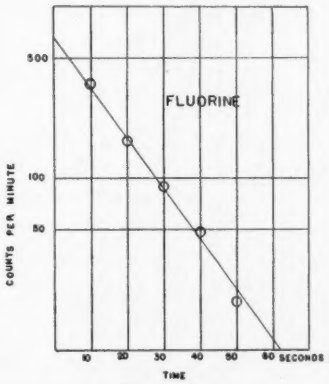

FIGURE 1

The large effects in the first three of the elements made it possible to study them in some detail. First of all, decay periods were observed. The time variation of the activity of the calcium fluoride (ascribable to fluorine because of the inappreciable activity observed in calcium), after exposure for one minute to the neutron radiation, is plotted in figure 1 with ordinates on a logarithmic scale representing rates of counting in counts per minute. It is seen that the half-life is very close to ten seconds, the value given by Fermi.*

The neutron-induced activity in aluminum, copper and silver was measured with an air ionization chamber and electroscope of the Lauritsen[8] type. The irradiated samples were placed just outside the ionization chamber next to a thin aluminum window. Observations of the ionization were made with various thicknesses of aluminum interposed between the activated substance and the ionization chamber. The results for silver are

shown in figure 2 where the observed ionization is plotted logarithmically (arbitrary scale) as a function of the time. With no absorber there is an initial rapid decay which turns off into a slower and constant value indicating two or more decay periods. As aluminum foils are interposed, the short period activity becomes relatively more important until with a thickness of 1 mm. of aluminum or more, the longer period group is quite inappreciable.

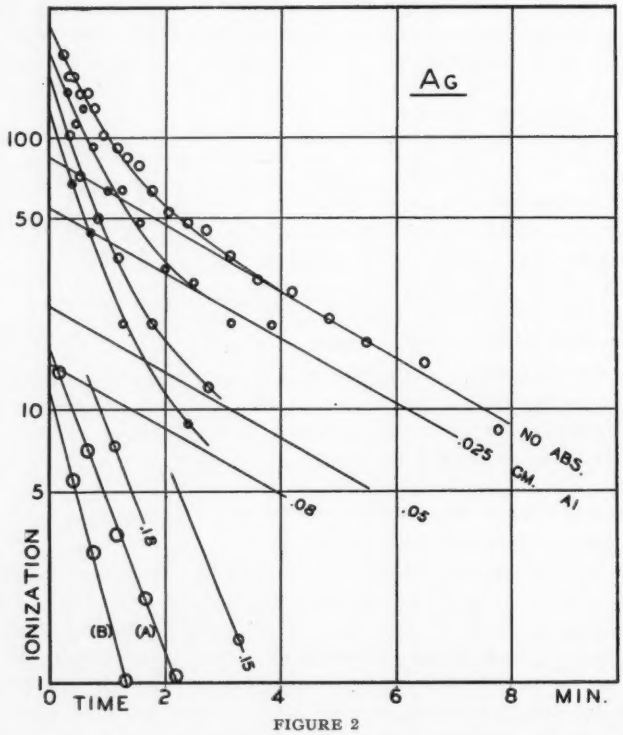

FIGURE 2

The observations have been analyzed on the assumption of two distinct decay periods. It seems reasonable to suppose that this is connected with the existence of two silver isotopes. From each curve the relative amount of long and short lived activity was estimated by a procedure involving the extrapolation to zero time of the longer period activity. Curves *A* and *B* accordingly represent the short period activity for no absorber and an absorber of 0.08 cm. of aluminum.

From these observations as well as others with the Geiger counter, the half-life of the longer period activity is estimated to be 2 minutes, 34 seconds

± 10 seconds, while the shorter period activity has a half-life of 26 ± 4 seconds. This is to be compared with Fermi's observation of a period for silver of about 2 minutes.

Corresponding results for copper and aluminum are shown in figure 3. In the case of aluminum the observed decay period is 9.5 ± 0.5 minutes, a value somewhat lower than that given by Fermi, i.e., 12 minutes. The observations on copper are not so accurate, and give an approximate decay

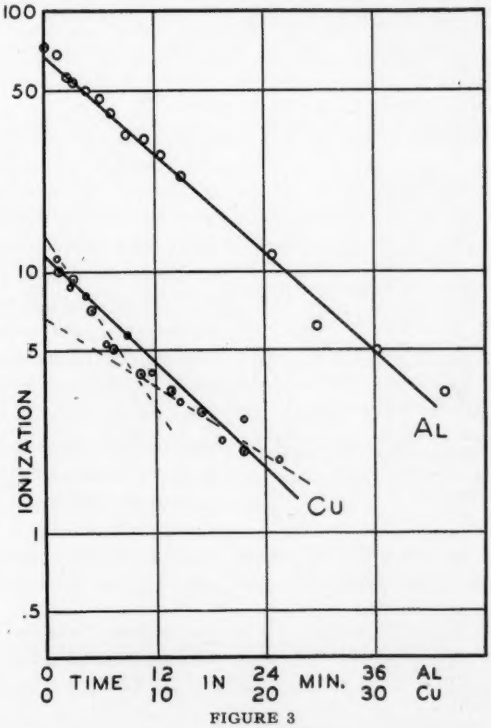

FIGURE 3

period of 7 minutes with a slight indication that actually there are two not very different periods of decay. This possibility is indicated in the figure by the dotted lines.

It was clear that the observed radiation in each case consisted in the main of particles of electronic mass, either electrons or positrons. By deflecting the particles in a magnetic field, we established that the particles from aluminum and both groups of particles from silver are electrons.*

Figure 4 is a plot of the relative intensity of the two kinds of activity in silver as a function of the absorber interposed between the active silver

and the ionization chamber or Geiger counter. The short life group is seen to be very much more penetrating than the other, the indications being that the maximum energy of the beta-particles of the former exceeds 3 MV, while that of the latter is something like 1.6 MV. Similar absorption measurements were made for copper and aluminum, with the results that the activities were reduced to a tenth by absorbers of 1 and 1.5 mm. of aluminum, respectively.

That there exists a relation between the energy of beta-particles and the disintegration probability, similar to the Geiger-Nuttall law for alpha-particles, has been pointed out by Sargent,[9] and it appears, in so far as the measurements go, that the Sargent relation applies as well to the beta disintegrations observed in our experiments.**

We have been able to estimate in absolute terms the activity induced in the substances by a given number of neutrons passing through, both from the Geiger counter measurements and the activity recorded by the electroscope. Probably the greatest uncertainity in these estimates of the nuclear collision cross-sections for induced radioactivity arises from the uncertainty of the absolute number of neutrons emanating from the beryllium target. From Dunning's[10] measurements of neutron collision cross-sections, we have taken as a basis for an estimate that for every one thousand neutrons passing through a paraffin block, one recoil proton at the back surface is detectable in our experiments.

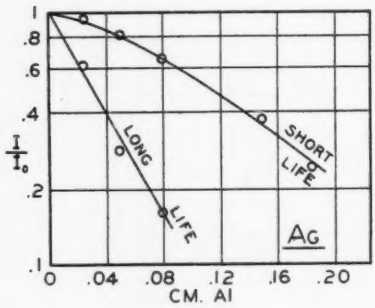

FIGURE 4

ELEMENT	HALF-LIFE (SECONDS)	APPROXIMATE RANGE OF DISINTEGRATION PARTICLES (CM. Al)	NUCLEAR CROSS-SECTION FOR ACTIVATION CM.^{2} ($\times 10^{26}$)	ACTIVATION CROSS-SECTION / COLLISION CROSS-SECTION
Ag (two groups)	26 ± 4	0.28	1.9	0.012
	154 ± 10	0.1	4.2	0.026
Al	570 ± 30	0.1	0.5	0.006
Cu	420 ± 60	0.15	0.3	0.002
F	10 ± 0.5	..	3.5	

The results are collected in the table above. The third column records the thickness of aluminum to reduce the observed activity to one-tenth. The fourth column gives the nuclear cross-section for activation, and the fifth column gives the ratio of the cross-section for activation to the cross-section for scattering as found by Dunning.

It is noteworthy that there is a very high probability in every case here studied that when a neutron makes a nuclear collision it will produce radioactivity and that therefore these reactions are not to be left out of consideration in interpreting observations on radioactivity induced by deuton bombardment. Calcium fluoride is a case in point.

We are much indebted to Dr. Edwin McMillan for making the electroscope measurements, and to Commander T. Lucci for his assistance throughout the experiments. We acknowledge with thanks a grant-in-aid of these experiments from the Josiah Macy Jr. Foundation, and the continued support of the Research Corporation and the Chemical Foundation.

[1] G. N. Lewis, M. S. Livingston and E. O. Lawrence, *Phys. Rev.*, **44**, 55 (1933).
[2] M. A. Tuve and L. R. Hafstad, *Ibid.*, **45**, 651 (1934).
[3] E. O. Lawrence and M. S. Livingston, *Ibid.*, **45**, 220 (1934).
[4] M. C. Henderson, M. S. Livingston and E. O. Lawrence, *Ibid.*, **45**, 428 (1934).
[5] E. Fermi, *La Ricerca Scientifica*, 5, **1**, No. 5 (1934).
[6] F. Joliot and I. Curie, *Nature*, **133**, 201 (1934).
[7] E. O. Lawrence and M. S. Livingston, *Phys. Rev.*, **45**, 608 (1934).
[8] H. R. Cane and C. C. Lauritsen, *Ibid.*, **45**, 226 (1934).
[9] B. W. Sargent, *Proc. Roy. Soc.*, **A139**, 659 (1933).
[10] J. R. Dunning, *Phys. Rev.*, **45**, 586 (1934).

* In a communication just received (E. Fermi, *La Ricerca Scientifica*, 5, **1**, No. 5, p. 28 (1934)) Professor Fermi finds for fluorine a somewhat shorter decay period. He has also shown that the disintegration particles from several substances including silver and aluminum are electrons.

** Anderson and Neddermeyer (*Phys. Rev.*, **45**, 653 (1934)) find that the distribution in energies of positrons from radioactivity induced in boron, carbon and aluminum by deutons fits the Sargent relation.

THE SPECTRAL SENSIBILITY OF AVENA

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Communicated June 19, 1934

Young oat (*Avena*) seedlings when exposed to light unilaterally respond by bending toward the light. The relation between the intensity of the light and the reaction time for the bending shows two parts which occur at different energy levels.[1] When the illumination is confined to the extreme tip of the coleoptile, only the low energy portion of the response curve is obtained; when the coleoptile illumination is confined to the basal portion, the high energy part appears. The two parts of the response curve may in this way be isolated and extended beyond the limits set by the conditions obtaining under total illumination. The anatomical separation of the two sections of the response curve indicates that there are two photosensory systems in the young oat. This is confirmed by the fact that different regions of the spectrum isolated with color filters produce relatively different effects on the two parts of the curve.[1] In order to specify the two photosensory systems, I have measured their spectral sensitivity in detail.

Etiolated seedlings of *Avena sativa* 8 to 12 mm. long were used. Light from a vertical ribbon filament lamp was focused on the slit of a monochromator and the plant so placed that a narrow monochromatic image of the slit fell on it. A "neutral" gelatin wedge controlled the intensity of the emergent beam. The lamp, lens and monochromator were together measured for energy distribution by photometric comparison with a calibrated monochromator and source. The wedge transmission was calibrated spectrophotometrically for each wave-length used. The temperature was maintained at 25°C. throughout each experiment. Exposures were for 10 seconds and were controlled by a stop-watch. The reaction time (time between the beginning of the exposure and the initiation of bending) was determined to the nearest minute by measuring, at one-minute intervals, the position of the apex of the shoot with a telescope horizontally displaced by a micrometer screw and then plotting the successive positions against time. During each measurement the plant was illuminated by a dim red light to which it is insensitive. To measure tip responses in those regions to which the basal portion of the coleoptile is more sensitive than the tip, all but 1.5 mm. of the tip was shielded from the light by a cylinder of tinfoil. The energies were kept below the point where an increase in reaction time occurs.[1]

The measurements, an average of 6 readings to a point, each reading on a different plant, are in figure 1. Curves *e*, *f*, *g*, *h* and *i* are pure tip curves obtained by shielding the bases of the coleoptiles; curves *j*, *k*, *l*, *p*, *q* and *r*

are pure base curves (naked coleoptiles) secured in spectral regions where the tip sensibility is lower than the base; curves *a* and *m*, *b* and *n*, *c* and *m* and *d* and *o* are double curves in spectral regions of lower base sensibility. Note that curve *m* passes through data from two spectral bands. I have assumed that the single points of curves *q* and *r*, and the two points of curve *h* pass through hyperbolas identical with those of the remainder of the data.

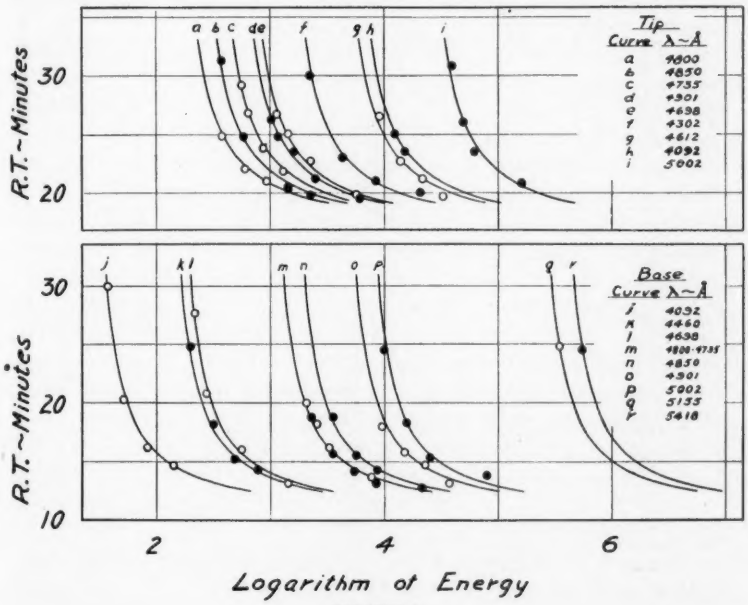

FIGURE 1

Relation between reaction time and log energy for responses of *Avena* to narrow spectral bands. Each circle is the average of six readings, each on a different plant. Open and solid circles are used alternately to avoid confusion.

The curves are all hyperbolas of the form $(r - m)(\log I + a) = k$ where m and a are the asymptotes of the reaction time and $\log I$ axes, respectively, r is the reaction time and k is a constant. The same equation describes the responses to white light.[1] The equation for the upper data is $(r - 18)(\log I + a) = 3.75$, and that for the lower $(r - 10.2)(\log I + a) = 3.35$, where a depends on the position of the curve on the $\log I$ axis.

The relative energy required at a given wave-length in order to produce a specific bending effect is given by the distance of its curve from the curve at the left of the figure which represents responses to the most effective light. The relative energies so obtained are given in figure 2. For obtain-

ing the vertical distance between these two curves, the energy required for a constant response by the tip and base for the same wave-length must be compared. Clearly the only part of the tip and base curves of figure 1, which may be regarded as comparable responses, are the thresholds. Hence, these were computed and the log I difference between them for each wave-length used to fix the relative position along the ordinates of the two curves in figure 2. The ordinates in figure 2, when read from top to bottom, give the logarithms of the relative energies. If read from bottom to top,

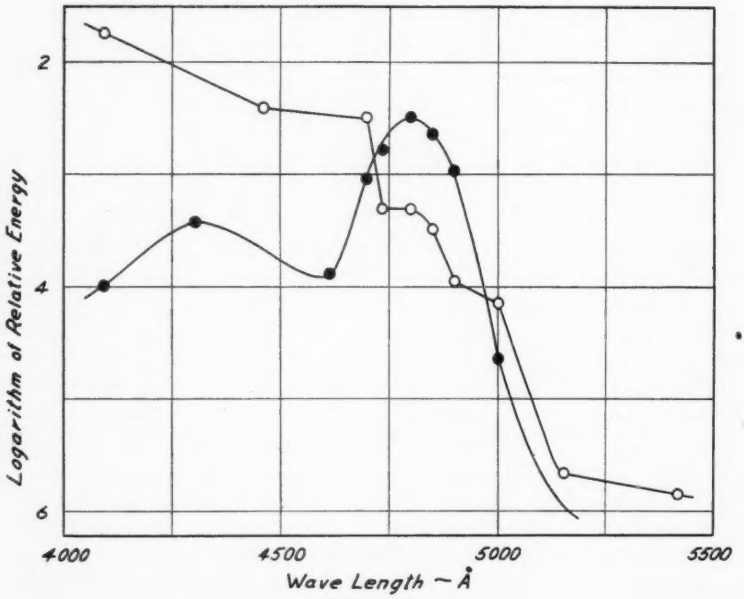

FIGURE 2

Spectral sensibility curves for phototropic responses of *Avena*. Open circles: base responses. Solid circles: tip responses.

they are the logarithms of the reciprocal of relative energy and therefore represent the relative effectiveness of the different parts of the spectrum.

The tip curve has a distinct maximum at λ 4800 and a secondary, very problematic one at about λ 4300. The base curve exhibits no maximum in the visible range. The high value at λ 4092 would seem to point to a maximum in the ultra-violet, but the reliability of the wedge calibrations at this wave-length is not great; hence, the high portions of the base curve may represent a broad maximum somewhere between λ 4000 and λ 4700. These results are in good agreement with those previously obtained with color filters.[1]

The absorption spectrum of a photochemical system is closely correlated with its chemical behavior. To what extent the spectral sensibility of a tropic reaction may approximate the absorption spectrum of the photic mechanism involved depends upon a variety of factors, e.g., the extent of masking by extraneous pigments, the solution state of the photic system, the optical conditions at the receptor. Except that pigment density in the coleoptile of *Avena* appears to increase slightly from the apex downward, principally after illumination, we have no reason to think that these conditions are not almost identical at the tip and base receptors. Therefore, the differences between the shapes of the two curves of figure 2 probably represent real differences in the absorption spectra of two photochemical systems.

Determinations of the relative effectiveness of different spectral regions on phototropic and photogrowth responses of *Avena* have been made by Blaauw,[2] Koningsberger,[3] Sonne[4] and Bachmann and Bergann.[5,6,7] Of these only Blaauw used a monochromator, with the sun and a carbon arc as sources. His use of Langley's Washington data for solar energy distribution introduces a large element of uncertainty into the measurements. Nevertheless his results and those of Koningsberger closely approximate sections of both curves of figure 2 as though the reactions observed were those of the tip or the base depending upon which possessed the greater sensitivity in a given spectral region. Bachmann and Bergann, by using small correction factors, succeeded in causing Blaauw's data to fall in line with their own and those of Sonne. This correction would also bring Blaauw's data closer to the base curve of figure 2 in the region λ 4700–5000 but would spoil the agreement above λ 5000.

The principal difficulty in comparing these various results is to decide whether the responses observed by each of these workers were those of the tip, of the base, or of both. Over most of the spectrum Blaauw and Koningsberger appear to have recorded base responses, and in the region λ 4750–5000 possibly tip responses. This is expected from the fact that they used naked plants. On the other hand Bachmann and Bergann and Sonne, also using naked plants, seem to have observed only base responses throughout.

[1] Haig, C., *Proc. Nat. Acad. Sci.*, **20**, 296–303 (1934).
[2] Blaauw, A. H., *Rec. trav. bot. neérl.*, **5**, 209 (1909).
[3] Koningsberger, V. J., *Ibid.*, **19**, 1 (1922).
[4] Sonne, C., *Stahlentherapie*, **31**, 778 (1929).
[5] Bachmann, F., *Planta*, **8**, 486 (1929).
[6] Bachmann, F., and Bergann, F., *Ibid.*, **10**, 744 (1930).
[7] Bergann, F., *Ibid.*, **10**, 666 (1930).

FURTHER EXPERIMENTS ON THE INHIBITION OF THE DEVELOPMENT OF LATERAL BUDS BY GROWTH HORMONE

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Communicated July 2, 1934

In earlier papers (Thimann and Skoog, 1933, 1934) it was shown that the inhibition of lateral buds by the terminal bud in *Vicia Faba* is due to the production of relatively large quantities of growth hormone by the latter. This substance passes basipetally through the stem, thus reaching the lateral buds, where it inhibits their growth. In these experiments it was shown that a suitable concentration of the hormone, if applied to decapitated plants, completely inhibits the development of lateral buds. The mechanism whereby a substance whose function is to promote growth may also inhibit it was interpreted in terms of the theory that in the presence of a high concentration of the growth hormone its production by a given tissue is prevented.

The present paper is concerned with two further points arising from the previous work. The first is in connection with the mechanism of the inhibition. It has been suggested by Laibach that our explanation that the growth substance inhibits bud development directly, on account of its excess concentration, is incorrect. He considers the inhibition to be only a secondary phenomenon accompanying an increased growth of the stem, which, according to him, takes place when growth substance is applied to the cut surface of the plant. Laibach's experiments are in many ways not strictly comparable with ours. He used living pollen grains as a source of growth hormone, and his paper implies that the plants were decapitated so close to the base that only the cotyledonary buds could develop. Under these circumstances he found that when pollen grains were applied to the decapitated stump a marked increase in growth, particularly in thickness, took place, and the outgrowth of the cotyledonary buds was retarded. In our experiments, on the other hand, a preparation of growth hormone obtained from *Rhizopus suinus*, and of known purity (about 5%) was used. The plants were grown until 3 or 4 leaves had developed; the terminal bud and youngest leaf were then removed and the growth of the lower lateral buds (Nos. 1 and 2 in Fig. 1) was determined. Given amounts of growth substance in agar, or of plain agar, were applied to the cut surface of the stem at regular intervals. Under these conditions cotyledonary buds did not develop. In all our experiments it was observed that (1) there was no increase in *thickness* of the stem in plants to which growth hormone was applied, and (2) as is shown in table 7 of the earlier paper (1934),

even when the concentration of applied growth hormone was sufficient to cause complete inhibition there was no increase in *length* of the stem over the controls. The reason for this, as was shown with defoliated plants, is that the amount of residual growth substance in the plant, and the amount synthesized by the green parts, is enough to supply all that can be utilized

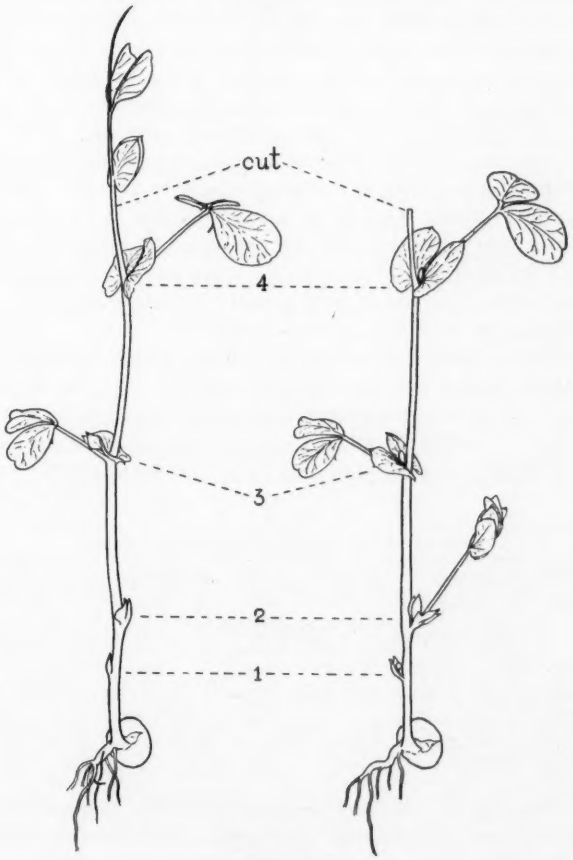

FIGURE 1

in stem elongation. Laibach's argument that the growth hormone concentration used in the above experiments was insufficient to show the growth-stimulating effect is therefore invalid. On the contrary, so far as growth-promotion is concerned only small amounts are necessary, and these are already present; for inhibition, the concentration of applied hor-

mone, like the concentration diffusing from the terminal bud, must be relatively high. As was concluded earlier, therefore, the inhibition of lateral buds cannot possibly be ascribed to any stimulation of growth.

The second point has to do with the possibility that the inhibition is due not to the growth hormone itself but to a special inhibitor present in our preparations. This seems improbable, since the inhibitor, if it were present in the purified preparation, must have been produced, together with the growth substance, both in the mould culture and in the tip of the experimental plants. Nevertheless the question of the existence of a special inhibitor must be settled before the explanation that the inhibition is due to the growth substance itself can be definitely accepted. If it could be shown that the completely pure growth substance inhibits as actively as the relatively impure preparation this point would be proved. Through the generosity of Professor Kögl we have now been able to test his crystalline preparations from urine and other sources (Kögl, Haagen-Smit and Erxleben, 1933) for their inhibiting activity. The present experiments show that these preparations are at least as active in inhibiting development of lateral buds as the solutions previously used.

Material and Methods.—It was found by Snow (1929) that the behavior of *Pisum sativum* with regard to bud development is very similar to that of *Vicia Faba*. Since peas are somewhat more convenient we have used them exclusively. The strain "Alaska," a pure line, was grown in the greenhouse until three leaves had developed and then decapitated so that two leaves

TABLE 1

INHIBITION BY CRYSTALLINE HETERO-AUXIN

Pisum: 20 plants, 2 weeks old; no leaves present; applications every eight hours for 6 days; mean stem length at start approx. equal; measurements of Bud No. 2 only.

	LENGTH OF BUD IN MM.			LENGTH OF STEM IN CM.
	DAY 0	DAY 4	DAY 6	DAY 6
Decapitated; water applied	<1.5	5.2 ± 0.8	9.0 ± 2.0	11.2 ± 0.3
Decapitated; 0.05 cc. of Hetero-auxin, 7000 units per cc. applied	<1.5	<1.5	<1.5	11.6 ± 0.3

remained (see Fig. 1). In the experiment of table 1 younger plants were used, in which only one leaf had developed, and this was removed with the terminal bud. The bud developing in the stipule below the oldest leaf (Bud No. 2) was principally observed. Small paraffin cups were moulded onto the cut surface of the stem and an aqueous solution of growth substance introduced into the cup. The cups were refilled every eight hours. The experiments were carried out in a humidity high enough to prevent rapid evaporation. This technique was found as satisfactory for our purposes as the application of agar blocks, and less time-consuming.

Results.—A preliminary experiment, using agar blocks containing Auxin A (auxentriolic acid) Auxin B (auxenolonic acid), Hetero-auxin, and the *Rhizopus* preparation Br.F.*, similar to the preparation used in our previous experiments, gave the following results: Auxin A, which had lost almost all its growth-promoting activity when received, produced no inhibition; Auxin B and Hetero-auxin, at a concentration of about 1000 units per cc., gave about the same inhibition as Br.F. at the same concentration.

The next experiment (table 1) proves that the crystalline compound is capable of causing complete inhibition without any increase of stem length. The buds less than 1.5 mm. long were not measured, since, in order to avoid any possible damage to them, the stipules were not removed.

Inhibition by the crystalline substance is thus complete, and this being proved, it only remains to show that for a given concentration in growth-promoting units, the crystalline compounds produce as great an inhibition as the impure preparation. Hence with the remaining available material the activities were standardized with *Avena* and adjusted to concentrations of 1000, 3000 and 5000 units per cc., and compared with Br.F. at the same concentrations. The results are given in table 2. At the start all buds were less than 1.5 mm. long. The experiments had to be discontinued after 5 days, and in three solutions after 4 days, owing to shortage of material. In this experiment buds other than No. 2 also developed to some extent, and their measurements are included in the table; it may be seen that they develop in the same ratio as the others, except that Bud No. 4 is relatively little affected.

TABLE 2

COMPARISON OF HORMONE PREPARATIONS AT DIFFERENT CONCENTRATIONS

Plants grown in light; all buds less than 1.7 mm. long at start.

		LENGTH OF NUMBERED BUD IN MILLIMETERS									
	BUD	DECAP.	AUXIN B		BR.F. (RHIZOPUS)			HETERO-AUXIN			INTACT
DAY	NO.	CONTROLS	3000	5000	1000	3000	5000	1000	3000	5000	CONTROLS
4	2	7.8	3.3	2.6	4.4	4.3	3.2	2.6	2.3	2.4	1.7
		±0.6	±0.3	±0.2	±0.6	±0.8	±0.8	±0.1	±0.1	±0.3	±0.1
5	2	14.0	4.8	4.4*	7.5	8.1	6.0	3.7	3.8*	4.5*	1.7
		±0.6	±0.8	±0.6	±1.5	±1.8	±1.5	±0.8	±0.3	±0.5	±0.1
5	1	2.6	1.1	1.2	1.2	1.2	1.2	1.0	1.8	1.0	no development
5	3	6.4	3.7	2.6	4.3	3.6	3.4	2.9	2.8	2.6	no development
		±1.2	±0.4	±0.2	±0.9	±0.3	±1.0	±0.4	±0.3	±0.5	
5	4	2.6	2.0	1.8	2.4	1.2	1.2	1.9	1.8	1.4	no development
No. of plants		15	10	7	9	7	7	11	5	5	15

* Application of hormone stopped at middle of 4th day.

On account of the small number of plants used the results do not show a smooth variation with concentration. However, it is clear that the two

crystalline compounds are, unit for unit of growth substance, at least as active as the *Rhizopus* preparation Br.F. The table further shows that the Hetero-auxin produces a more complete inhibition than the other two substances. This is in accordance with the fact that while the latter preparations steadily decrease in activity on keeping, Hetero-auxin remains constant, and therefore retains its full activity throughout the experiment. Further, Hetero-auxin is much less readily attacked by oxidizing agents than the other compounds and therefore will be less inactivated in the plant. That this loss in activity may be considerable has been shown by inactivation with crushed plant tissue (Thimann, 1934).

On account of the possibility that the plants might be permanently injured by the application of high concentrations of growth hormone, the buds were re-measured 9 days after the close of this experiment. As a rule Bud No. 2 at first develops the most rapidly, but in a few cases one of the other buds may subsequently overtake it. When this happens, the more rapidly growing bud may completely stop the growth of Bud No. 2, as has been previously discussed (Thimann and Skoog, 1934). In order to determine the total amount of bud elongation, therefore, it was necessary to add together the final lengths of buds 1, 2, 3 and 4. The mean total bud lengths in mm. on the 14th day after decapitation were, for the various concentrations of growth hormone, as follows:

Decapitated	Auxin B		Hetero-auxin			Br. F.			Intact
controls	3000	5000	1000	3000	5000	1000	3000	5000	controls
184	174	169	117	155	120	156	194	185	<4

The figures show that the inhibited buds grow in a normal manner after the application of hormone has ceased. The buds are now equal in length to those on the decapitated controls in all but the Hetero-auxin series. The latter are still somewhat behind the others; the buds are, however, rapidly growing, and the difference is to be ascribed to the greater stability of the Hetero-auxin, as discussed above.

Conclusions.—The inhibition of lateral buds brought about by the application of growth substance after decapitation is not due to a stimulation of the growth of the experimental plants, since inhibition is complete without any accompanying increase in stem length or thickness. The application of two different crystalline preparations of growth substance, auxin B and Hetero-auxin, to decapitated *Pisum* shows these to be at least as active in causing inhibition as the impure preparation from *Rhizopus*, when used in the same growth substance concentrations. A third crystalline compound, auxin A, which had lost most of its growth-promoting activity produced no inhibition. The inhibition produced by the application of the growth hormone is not accompanied by any injury to the plant.

* Activity 12,000 units per mg. = $2.4.10^6$ plant units per mg. = 5% pure growth substance.

Kögl, F., Haagen-Smit, A. J., and Erxleben, H., *Z. Physiol. Chem.*, **216**, 31 (1933).
Laibach, F., *Ber. d. Deutschen Bot. Ges.*, **51**, 336 (1933).
Snow, R., *New Phytologist*, **28**, 345 (1929).
Thimann, K. V., *J. Gen. Physiol.* (in press).
Thimann, K. V., and Skoog, F., these PROCEEDINGS, **19**, 714 (1933).
Thimann, K. V., and Skoog, F., *Proc. Roy. Soc.*, **B114**, 317 (1934).

THE POTENTIAL OF A POSITIVE MASS AND THE WEIGHT FUNCTION OF WIENER

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Communicated July 3, 1934

The Potential of a Positive Mass.—Let $\mu(e)$ be a completely additive non-negative function of sets measurable Borel in three dimensions. The integral

$$u(M) = \int_W \frac{1}{MP}\, d\mu(e_p),$$

where the region of integration is all of space, is defined as $\lim_{n=\infty} \int_W \varphi_n(M,P)\, d\mu(e_p)$; $\varphi_n(M,P) = 1/MP$ if $1/MP \leq n$, and is equal to n otherwise. The function $u(M)$ is called the potential of the mass distribution $\mu(e)$.

In the sequel it will be assumed that $\mu(e) = 0$ for all e measurable Borel outside some sufficiently large sphere, and that $u(M)$ is bounded.

A point P will belong to the set F if and only if $\mu(e) > 0$ for every sphere σ with center P; e is the set of points on and interior to σ. Obviously F is closed. We should then have

$$u(M) = \int_F \frac{1}{MP}\, d\mu(e_p).$$

This follows from the fact that for a fixed M the integral is an additive function of sets.[1]

The volume average

$$A_n(P_0) = \frac{3n^3}{4\pi} \int u(P)dv$$

of $u(M)$, where the integral is taken over the volume of the sphere with center P_0, and radius $1/n$, is continuous and for fixed P an increasing function of n; furthermore $\lim_{n=\infty} A_n(P) = u(P)$.[2]

It follows from the lemma of this note that the upper bound of $u(M)$ for points which lie in a domain Σ which contains no points of F, but whose boundary t lies in F, cannot exceed the upper bound of $u(M)$ for points in t. This was conjectured by G. C. Evans,[3] and is of use in showing that various definitions of capacity are equivalent. We shall also study the relation between the weight function defined by Wiener[4] in his treatment of the Dirichlet problem and the sweeping out of the unit mass.

LEMMA. *Let $u_n(M)$, M in CF (the complement of F), be the sequence solution of the Dirichlet problem for boundary values $A_n(P)$ on t, the boundary of the largest open connected piece Σ containing M which has no points in common with F. Then $\lim_{n=\infty} u_n(M) = u(M)$ for every M in CF.*

If F contains an interior point, t is of positive capacity. In fact, the capacity of t would not be less than the capacity of some sphere. On the other hand, if F contains no interior point, t can be of capacity zero. If t is of capacity zero, then M must be a point of the single piece Σ extending to infinity. In this case the capacity of F is zero and the lemma is obvious, since $u(M)$ would be identically zero.

The sequence solution for boundary values $A_n(P)$ is given by

$$u_n(M) = \int_t A_n(P)dm(e_p; M),$$

where $m(e; M)$ is the mass on the subset e of t, obtained by sweeping out the unit mass at M onto t. For sufficiently smooth boundaries this result has been proved.[5] The general conclusion follows from the weak convergence of the distributions of mass obtained by sweeping out the unit mass onto the approximating boundaries and the uniqueness of the mass distribution $m(e; M)$.

Since $A_n(P) \leq A_{n+1}(P) \leq u(P)$ and $\lim_{n=\infty} A_n(P) = u(P)$, it follows that

$$\bar{u}(M) = \lim_{n=\infty} u_n(M) = \int_t u(P)dm(e_p; M).$$

We may also write

$$\bar{u}(M) = \int_F u(P)dm(e_p; M),$$

since $m(e; M) = 0$ if e is in the complement of t. Therefore

$$\bar{u}(M) = \int_F \left[\int_F \frac{1}{SP} d\mu(e_s) \right] dm(e_p; M). \qquad (1)$$

Since (1) exists we may change the order of integration. We shall then

have

$$\bar{u}(M) = \int_F \left[\int_F \frac{1}{SP} dm(e_p; M) \right] d\mu(e_s). \qquad (2)$$

Consider now

$$v(S; M) = \int_F \frac{1}{SP} dm(e_p; M)$$

$v(S; M)$ is the potential at S of the mass obtained by sweeping out onto t the unit mass at M. If S is a regular point of t or is in the complement of $t + \Sigma$, this potential is $1/MS$.[6] The mass $\mu(e)$ for the set e of irregular points is zero. In fact, if $\mu(e) > 0$, then e would contain a closed set e_c for which $\mu(e_c) > 0$. This set would be of positive capacity and would therefore contain a regular point.[7]

It follows that $\bar{u}(M) = u(M)$. We have seen that $\mu(e) = 0$ for the set e for which $v(S;M) \neq 1/MS$. The value of (2) will hence be unchanged if $1/MS$ is integrated with respect to $\mu(e)$.[8]

THEOREM. *Let Σ be a domain, that is, a connected open set, which contains no point of F, but whose boundary t lies in F. Then the upper bound of $u(M)$, u.b. of $u(M)$, for M in Σ is not greater than u.b. $u(M)$, M in t.*

Consider any point M_1 in Σ, then $u_n(M_1) \leq A_n(M_0) \leq u(M_0)$ for some M_0 in t. This is a property of the sequence solution. Now $u(M_0) \leq$ u.b. $u(M)$, M in t. Hence

$$u(M_1) = \lim_{n=\infty} u_n(M_1) \leq \text{u.b. } u(M), M \text{ in } t.$$

This proves the theorem.

COROLLARY. *The upper bound of $u(M)$ at a point P of t for points in Σ cannot exceed the upper bound of $u(M)$ at P for points in t.*

Define $u_{G \cdot F}(M)$ as the potential due to the mass on $G \cdot F$. Let $l_1(r) =$ u.b. $u_{\sigma(r) \cdot F}(M)$, M in $t \cdot \sigma(r)$, where $\sigma(r)$ is the set of points whose distances from P do not exceed r. Similarly, let $l_2(r) =$ u.b. $u_{\sigma(r) \cdot F}(M)$, M in $\Sigma \cdot \sigma(r)$. Then $\lim_{r=0} l_1(r) = l_1$, and $\lim_{r=0} l_2(r) = l_2$ exist. From the preceding theorem $l_1 \geq l_2$. Further $u_{(C\sigma(r)) \cdot F}(M)$ is continuous for M in $\sigma(r_1)$, $r_1 < r$. It is then easy to see that the difference of the two upper bounds at P is precisely $l_1 - l_2$.

The Weight Function and the Sweeping Out of the Unit Mass.—Let Σ be a domain in three dimensions with bounded boundary t. Let e_1 and e_2 be two distinct closed subsets of t. The weight $W_{e_1 e_2}(e_1; P)$ of e_1 at P in the system e_1, e_2 is defined as the value at P of the sequence solution determined by boundary values 1 on e_1 and 0 on e_2, and which vanishes at infinity if the domain whose boundary is $e_1 + e_2$ extends to infinity. The lower bound of $W_{e_1 e_2}(e_1; P)$, e_1 fixed and e_2 a closed subset of t having no

points in common with e_1, is called the outer weight, $W(e_1; P)$, of e_1 at P. Of course $W(e_1; P)$ might be zero. This function is defined only for closed sets.

A unit mass at P, a point of Σ, can be swept out onto the boundary t of Σ so that the potential of this swept out mass $m(t; P)$ is the same as the potential of the unit mass at points in $C(\Sigma + t)$ and at regular points of t.

THEOREM. *$W(e_1; P) = m(e_1; P)$ for every closed set e_1 of t and every P in Σ.*

Let P be a point in Σ and e_1 a closed subset of t. e_1 is contained in an open set 0 such that $m(0 - e_1; P)$ is arbitrarily small, say $< \epsilon$. Now $t - 0 \cdot t$ is a closed set, say e_2. Sweep now the mass $m(t - (e_1 + e_2); P)$ onto $e_1 + e_2$. Denote by $\bar{m}(e; P)$ this new mass distribution. Since the total mass is not increased by sweeping out it follows that

$$0 \leq \bar{m}(e_1; P) - m(e_1; P) < \epsilon,$$

$$0 \leq \bar{m}(e_2; P) - m(e_2; P) < \epsilon.$$

We wish to show now that $\bar{m}(e; P)$ is the same for any set measurable Borel e in $e_1 + e_2$ as the mass $\bar{\bar{m}}(e; P)$ on e obtained by sweeping out the unit mass at P onto $e_1 + e_2$. A regular point S of $e_1 + e_2$ is a regular point of t. The potential of S due to the mass distribution $m(e; P)$ is continuous at S, equal to $1/SP$.[9] It follows that the potential at S due to the mass distribution $\bar{m}(e; P)$ is $1/SP$. Also the potential at S due to the mass distribution $\bar{\bar{m}}(e; P)$ is $1/SP$.

The potential $\bar{u}(M)$ due to the mass distribution $\bar{m}(e; P)$ is a function harmonic and bounded in $C(e_1 + e_2)$, which takes on at every regular boundary point S the value $1/SP$. This function is unique.[10] Therefore $\bar{\bar{u}}(M)$, the potential due to the mass distribution $\bar{\bar{m}}(e; P)$, is equal to $\bar{u}(M)$ for all points in $C(e_1 + e_2)$ and at all regular boundary points of $e_1 + e_2$. Then $\bar{u}(M)$ is equal to $\bar{\bar{u}}(M)$ for all points M. It follows that the corresponding mass distributions are identical on all sets measurable Borel, that is, $\bar{m}(e; P) = \bar{\bar{m}}(e; P)$.

Consider now two closed sets e_1 and e_2 of t, $e_1 \cdot e_2 = 0$. $\bar{m}(e_1; P)$ is the value at P of the sequence solution determined by boundary values 1 on e_1 and 0 on e_2. Therefore

$$W(e_1; P) \leq W_{e_1 e_2}(e_1; P) = \bar{m}(e_1; P) < m(e_1; P) + \epsilon$$

for arbitrary $\epsilon > 0$. This proves that $W(e_1; P) \leq m(e_1; P)$. On the other hand,

$$m(e_1; P) \leq \bar{m}(e; P) = W_{e_1 e_2}(e_1; P) < W(e_1; P) + \epsilon$$

for arbitrary $\epsilon > 0$. This gives $m(e_1; P) \leq W(e_1; P)$. The two inequalities prove the theorem.

[1] J. Radon, "Theorie und Anwendungen der absolut additiven Mengenfunktionen," *Wiener Sitzungsberichte*, **122**, Abt IIa, July, 35 (1913).

[2] F. Riesz, "Sur les fonctions subharmoniques et leur rapport à la théorie du potential," *Acta Math.*, **54**, 345 (1930).

[3] G. C. Evans, Seminar lectures given at Rice Institute, 1933–1934. Referred to as SE.

[4] N. Wiener, "Certain Notions in Potential Theory," *Jour. Math. Phys.*, M.I.T., **3**, No. 1, 28 (January, 1924).

[5] C. de la Vallée Poussin, "Extension de la méthode du balayage de Poincaré et problème de Dirichlet," *Ann. de l'Inst. H. Poincaré*, **2**, 205 (1932).

[6] G. C. Evans, SE.

[7] G. C. Evans, "Application of Poincaré's Sweeping-Out Process," these PROCEEDINGS, **19**, 457–461 (1933).

[8] J. Radon, *Ibid.*

[9] G. C. Evans, SE.

[10] O. D. Kellogg, *Foundations of Potential Theory*, Berlin, 335 ff., and G. C. Evans, loc. cit., 460 (1929).

V_3 IN R_6

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Communicated July 10, 1934

I. *Introduction.*—In this paper, we shall summarize the results of studies on particular V_3 in R_6. The fundamental ideas of this work can be found in a doctor's thesis "A Study of 3-Dimensional Riemann Surfaces in 6-Dimensional Euclidean Space," written by the author under the direction of Professor D. J. Struik. The other results which will be found in this summary are from unpublished papers by the same author.

We shall use the general notation of Eisenhart as was developed in "Riemannian Geometry."

The basis of our work is Eisenhart's existence theorem. "Given an R_6; $\underset{pq}{v_a}$, $\underset{p}{h_{ab}}$, g_{ab} $(a,b = 1, 2, 3)(p,q = 4, 5, 6)$ satisfying the Gauss, Codazzi and Ricci Relations then a V_3 in R_6 is determined."

We shall use $y^\lambda(\lambda = 1..6)$ as the Euclidean orthogonal coördinates mapping R_6; $u^a(a = 1, 2, 3)$ as the intrinsic V_3 coördinates. The theorem of Eisenhart implies that $\underset{pq\ p}{v_a h_{ab} g_{ab}}$ are given as functions of (u^a). However, we shall classify

(A) $\underset{pq}{v_a}$ as to value

(1) $\underset{pq}{v_a} = 0$

(2) $v_{45a} = 0, v_{46a} = 0, v_{56a} = ?$

(3) $v_{45a} = 0, v_{46a} = ?, v_{56a} = ?$

(B) $h_{\overset{p}{ab}}$ as to rank

(1) $h_{\overset{p}{ab}}$ of rank (1)

(2) $h_{\overset{4}{ab}}$ of rank (1); $h_{\overset{5}{ab}}$ of rank (1); $h_{\overset{6}{ab}}$ of rank (2)

(3) $h_{\overset{4}{ab}}$ of rank (1); $h_{\overset{5}{ab}}$ of rank (2); $h_{\overset{6}{ab}}$ of rank (2)

and then determine these quantities as functions of u^a satisfying the Gauss, Codazzi and Ricci Relations for V_3 in R_6. We use the bracket [A,B] to signify some set of V_3 (as [1,2]).

We first study the case [1,1]

II. *V_3 in R_6 of* [1,1]. Let the vectors i_{1a}, i_{2a}, i_{3a} be arbitrary in V_3 then

$$h_{4ab} = h_{411} i_{1a} i_{1b}; \quad h_{5ab} = h_{522} i_{2a} i_{2b}; \quad h_{6ab} = h_{633} i_{3a} i_{3b} \tag{1}$$

$$v_{\overset{pq}{a}} = 0$$

The equations of Gauss, Codazzi and Ricci for V_3 in R_6 are

$$K_{abcd} = \Sigma_{p}(h_{\overset{p}{ac}} h_{\overset{p}{bd}} - h_{\overset{p}{ad}} h_{\overset{p}{bc}}) \tag{2a}$$

$$h_{\overset{p}{a}}[b,c] = \Sigma_{q} v_{\overset{pq}{a}}[b h_{\overset{q}{c}}] \tag{2b}$$

$$v_{\overset{pq}{a}}[a,b] = h_{\overset{p}{c}}[a h_{\overset{q}{b}}]c - \Sigma_{r} v_{\overset{pr}{a}}[a v_{\overset{rq}{b}}] \tag{2c}$$

Using (1), from the Gauss Relation (2a), we see for [1,1] that

$$V_3 \equiv R_3. \tag{3}$$

From (1), and the Ricci Relation (2c), we find that [1,1] contains (3) subcases according as

$$i_{1a} = i_{2a} = i_{3a} \tag{4a}$$

$$i_{1a} = i_{2a}; \quad i_{3a} \text{ is } \perp \text{ to } i_{1a} \tag{4b}$$

$$i_{1a}, i_{2a}, i_{3a} \tag{4c}$$

are mutually perpendicular. We denote these subcases by $[1,1]_a$; $[1,1]_b$; $[1,1]_c$ according as (1), (4a); (1), (4b); (1), (4c) are satisfied.

III. V_3 *of* $[1,1]_a$. For the normal curvature locus at any point P of V_3 we find

the normal curvature vectors lie on a straight line through P-"axial points." (1a)

All directions in the R_2 at P which is perpendicular to $\underset{1}{i}$ are asymptotic.

No other asymptotic directions exist. (1b)

From the Codazzi Relations (II, 2b) and (1), we find

V_3 contains ∞^1 flat R_2 cutting the ∞^2 curves of congruence $\underset{1}{i}$ orthogonally. (2a)

$\underset{1}{i}$ is the single principal direction of $\underset{p}{h_{ab}}$ (2b)

If $\underset{2}{i}, \underset{3}{i}$ are any two directions in the R_2 perpendicular to $\underset{1}{i}$ at P then $(\underset{2}{i}, \underset{3}{i})$ are flat R_2 building; $(\underset{1}{i}, \underset{3}{i})$; $(\underset{1}{i}, \underset{2}{i})$ are V_2 building. (2c)

In order to find the g_{ab} we study the Gauss Relation (II, 2 a) using an intrinsic coördinate system (u^a) with coördinate lines along the three mutually perpendicular vectors $\underset{a}{i}$. We find

$$ds^2 = \overline{[1 + u^2\theta_2(u^1) + u^3\theta_3(u^1)]du^1}^2 + \overline{du^2}^2 + \overline{du^3}^2 \qquad (3)$$

where θ_2, θ_3 are arbitrary functions of (u^1).

Since this V_3 has only axial points, we are unable to determine from the normal curvature properties whether V_3 lies in R_6 and not in R_4 nor R_5. By a study of rotations of the normal vectors to V_3 and by means of Eisenhart's theorem of (I), we are enabled to find a necessary and sufficient criterion that V_3 lie in R_6 and not in R_4 nor in R_5. An application of this test shows that V_3 of $[1,1]_a$ lie in R_6 and not in R_4 nor R_5.

We are interested in a geometric method of generating V_3 of $[1,1]_a$. Our method is to analyze the results of the Codazzi Relation which are *necessary* conditions that V_3 be of $[1,1]_a$ and pick those of the results which are *sufficient* to characterize $[1,1]_a$. The theorem which summarizes our results in this direction is the following:

"All the developable V_3 in R_6 which can be generated by ∞^1 flat R_2 are those of $[1,1]_a$. Conversely, every V_3 of $[1,1]_a$ is developable and can be generated by $\infty^1 R_2$ such that consecutive R_2 intersect. These V_3 are the envelopes of ∞^1 osculating R_3 to a V_1 in R_6 and conversely. This is valid for V_3 in R_n."

We next study $[1,1]_b$.

IV. V_3 *in* R_6 *of* $[1,1]_b$. Here, we have

$$\underset{4}{h_{ab}} = h\,\underset{4}{i}_a\,\underset{1}{i}_b;\quad \underset{5}{h_{ab}} = h\,\underset{5}{i}_a\,\underset{1}{i}_b;\quad \underset{6}{h_{ab}} = h\,\underset{6}{i}_a\,\underset{2}{i}_b \qquad (1)$$

$$v_{\underset{pq}{a}} = 0$$

$\underset{1}{i}$ is perpendicular to $\underset{2}{i}$.

Select $\underset{3}{i}$ perpendicular to $\underset{2\ 1}{i, i}$.

For the normal curvature locus at any point P of V_3 we find

The normal curvature vectors all lie in an R_2 through P (2a)

$\underset{3}{i}$ is the only asymptotic direction. (2b)

From the Codazzi Relation and (2), we find

V_3 has $\infty^2 R_1$ along $\underset{3}{i}$ direction (3a)

The $\underset{1\ 2}{i, i}$ which are perpendicular to $\underset{3}{i}$ are V_2 building. (3b)

$\underset{1\ 2\ 3}{i, i, i}$ are V_2 building in pairs. (3c)

Using curvilinear coördinates along $\underset{1\ 2\ 3}{i, i, i}$ we find from the Gauss Relation

$$ds^2 = \overline{h_1 du^1}^2 + \overline{h_2 du^2}^2 + \overline{h_3 du^3}^2;\ h_3 = 1 \tag{4a}$$

$$h_1 = a + bu^3;\ h_2 = c + du^3 \tag{4b}$$

$$\underset{1}{c} = \alpha a;\ \underset{1}{d} = \alpha b \tag{4c}$$

$$\underset{2}{a} = \beta c;\ \underset{2}{b} = \beta d \tag{4d}$$

$$\underset{1}{\alpha} + \underset{2}{\beta} + bd = 0. \tag{4e}$$

Here $a, b, c, d, \alpha, \beta$ are functions of (u^1, u^2) and subscripts on these letters indicate differentiation with respect to (u^1, u^2). To determine the meaning of (4), we study the family of ∞^1 parallel $V_2(\underset{1\ 2}{i, i})$ in $V_3 \equiv R_3$.

We find that on $u^3 = 0$

$$a = E^{1/2},\ c = G^{1/2}. \tag{5}$$

Furthermore, (4c,d) are the Codazzi Relations for this $V_2(u^3 \equiv 0)$. The condition (4e) means that the linear element of this V_2 is

$$ds^2 = \overline{(a du^1)}^2 + \left(\frac{\overline{du^2}}{a}\right)^2 \tag{6}$$

where u^1, u^2 are along the directions of principal curvature of V_2.

Finally if

$\lambda \equiv$ principal curvature along $u^1 \longrightarrow$ var (7)

$\sigma \equiv$ principal curvature along $u^2 \longrightarrow$ var

then

$$h_1 = a(\lambda u^3 - 1); \; h_2 = \frac{(\sigma u^3 - 1)}{a}. \tag{8}$$

Here as in $[1,1]_a$ we investigate the sufficient conditions that V_3 be of $[1,1]_b$. Examining the Codazzi Relations, we find by use of the Weingarten Formulae, that V_3 of $[1,1]_b$ can be generated as follows:

(a) Select a V_2 in R_6 with

(1) u^1, u^2 along the lines of principal curvature (9)

(2) Linear curvature characteristics $\left|\begin{smallmatrix} \lambda & \\ & \sigma \end{smallmatrix}\right|$

(3) $ds^2 = (adu^1)^2 + \left(\frac{du^2}{a}\right)^2$.

(b) Determine a V_2 in R_3 such that

(1) u^1, u^2 are along the lines of principal curvature

(2) $ds^2 = (adu^1)^2 + \left(\frac{du^2}{a}\right)^2$.

I.e., map V_2 in R_6 upon V_2 in R_3

(c) In this V_2 *in* R_3, determine the *principal curvatures* λ,σ

(d) On V_2 in R_6 of (a) draw congruences $\underset{3}{i}$ such that

$$\underset{3}{i} \text{ is } V_2 \text{ normal}$$

$$\frac{\partial \underset{3}{i}^\lambda}{\partial u^1} = -\lambda \frac{\partial y^\lambda}{\partial u^1}$$

$$\frac{\partial \underset{3}{i}^\lambda}{\partial u^2} = -\sigma \frac{\partial y^\lambda}{\partial u^2}.$$

The congruences $\underset{3}{i}$ thus determined generate V_3 in R_6 of $[1,1]_b$ and conversely.

Though this V_3 has planar points, we can prove that it lies in R_6 and not in R_4 nor R_5.

Finally, we shall briefly discuss $[1,1]_c$.

V. *V_3 in R_6 of* $[1,1]_c$. Proceeding as in the previous, we find

$$\underset{1}{i}, \underset{2}{i}, \underset{3}{i} \text{ are geodesics in } V_3 \tag{1a}$$

i, i, i are geodesic V_2 building when grouped in pairs. (1b)
$1\ 2\ 3$

Referred to the (u^a) system along i_a

$$ds^2 = \overline{du^1}^2 + \overline{du^2}^2 + \overline{du^3}^2. \tag{2}$$

Since the normal curvature locus lies in a plane R_2 which does not pass through the point P under consideration, no asymptotic directions exist. Hence *no straight lines exist upon this* R_3. Its integral equations are of the form

$$y^\lambda = \phi_1^\lambda(u^1) + \phi_2^\lambda(u^2) + \phi_3^\lambda(u^3). \tag{3}$$

The case [1,2] will be discussed in a future paper.